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NOTE

Technical note: relation between the Spencer-Attix-Nahum cavity integral and absorbed dose

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Abstract

Objective. In high energy photon beams, the Spencer–Attix–Nahum (SAN) cavity integral can play an important role for the determination of absorbed dose in a medium of interest using a detector. This is due to the fact that the SAN cavity integral may serve as a substitute for absorbed dose. In order to justify this substitution, the relation between absorbed dose and the SAN cavity integral must be precisely known. *Approach*. The formalism for the SAN cavity integral consisting of two terms, a fluence term and a track-end term, was reviewed. The commonality to the quantity of restricted cema was emphasized. Since the SAN cavity integral is considered a dosimetrical quantity, the tool of Monte Carlo (MC) dose calculation is adopted to obtain numerical values for both, absorbed dose and SAN cavity integral. The codes FLURZ and DOSRZ from the EGSnrc system were used for that along with improving certain approximations made in FLURZ. *Main Results*. The relation between the SAN cavity integral and absorbed dose was exactly formulated and numerically reproduced by the MC calculation. Using a comprehensible example it is confirmed that there is a very high level of agreement between MC calculated SAN cavity integral and MC calculated absorbed dose, and that this applies independently from the choice of cutoff values used in the MC calculation. *Significance*. The theory which is dealing with the conversion of the absorbed dose in the sensitive volume of a detector into the dose without the detector in place is generally known as cavity theory. The well-founded substitution of absorbed dose by the SAN cavity integral can provide useful additional applications of this theory.

1. Introduction

In dosimetry, the term "Cavity Integral" is used to describe the fact that certain dosimetrical quantities can be calculated as integrals over the product of radiometric quantities (such as the particle fluence differential in energy) and interaction coefficients (ICRU 2011). Here we are considering the so-called Spencer–Attix cavity integral. This type of cavity integral goes back on the well-known paper by Nahum (1987) on Monte Carlo (MC) calculated stopping power ratios. Although this name was not directly used in his paper, the underlying concept based on the Spencer–Attix approach (Spencer and Attix 1955) and referring to the fluence of electrons is usually attributed to Nahum (Kumar *et al* 2016, Andreo *et al* 2017). We therefore will denote it as 'Spencer–Attix–Nahum (SAN) cavity integral'.

Interestingly, Kellerer *et al* (1992) suggested fourteen years later the definition of a new dosimetrical quantity named 'reduced cema' by applying exactly the same concept. However, they did not mention the commonality of the ideas underlying this new quantity and the SAN cavity integral. Kellerer's suggestion for the new quantity was subsequently adopted by the ICRU (1998, 2011, 2016) known as 'restricted cema'.

Central in cavity theory is the ratio between the absorbed dose in the sensitive volume of a detector, D_{det}, and the absorbed dose at the position of its reference point in the undisturbed (i.e. without

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detector) medium, D_{med}. The determination of the absorbed dose D_{med} by a measurement is based on this ratio. Details are not given here since they are well described in relevant textbooks, for example, in FIORD (Andreo *et al* 2017). Within the framework of cavity theory, the relation between the SAN cavity integral and absorbed dose becomes an interesting topic: in case that this relation is known, the central ratio of cavity theory, D_{det}/D_{med} can be substituted by a ratio of cavity integrals with the potential of an alternative access to applications of cavity theory (Hartmann *et al* 2021).

Kumar *et al* (2016) presented calculated data of the SAN cavity integral in a 1 MeV photon beam in order to study their relation to absorbed dose, both obtained by MC calculations. They found a close equality between them. Comparisons were also made in dependence of the energy cutoff parameter Δ involved in the expression for the cavity integral. In this case, however, it turned out that the ratio between the SAN cavity integral and absorbed dose obtained under identical geometrical conditions increasingly deviates from one for larger values for the energy cutoff parameter Δ (larger than of about 20 keV).

In this paper we want to point out that a comparison between MC calculated cavity integral and dose should be performed with a numerical exactness as high as possible. Kumar *et al* (2016) have used the codes FLURZ and DOSRZ from the EGSnrc MC system (Rogers *et al* 2025). These codes, however, are using some approximations which are influencing particularly the numerical outcome of the SAN cavity integral. By improving such approximations, the relation between the SAN cavity integral and absorbed dose can be quantified more exactly. It can also be shown that this relation remains independent from the energy cutoff parameter Δ.

2. Methods

2.1. Principal considerations

The SAN cavity integral can be considered to represent quasi a dosimetrical quantity similar to the quantity of restricted cema (ICRU 2016). However, whereas the latter quantity refers according to its definition to a mathematical point, the purpose of the SAN cavity integral is to determine the total energy dissipation in an extended volume of interest due to electronic interactions. These are the interactions of the electrons with the shell electrons of the atoms of a medium in that volume.

It can be expected that under identical conditions the value of the cavity integral in a given volume should be very close to mean absorbed dose in that volume, particularly under the condition that those energy contributions to absorbed dose which are not due to electronic interactions are neglected. This must be the case if the MC method is applied for the calculation of both, the SAN cavity integral and absorbed dose. The reason is that the MC calculation of dose is also performed using the Spencer–Attix concept (Spencer and Attix 1955). This particularly means that the calculation of dose is based on dividing the energy distribution of electrons into two groups, into a high energy group and into a low energy group separated by the cutoff parameter Δ, and treating energy losses in these two groups in a different manner.

Applying the Spencer–Attix concept, the SAN cavity integral, CI_{SAN}, also consists of two components: a fluence based term denoted as FT and a track-end term, TE:

$$CI_{SAN} = FT + TE. \tag{1}$$

During the calculation of total absorbed dose D^{MC} in a volume of interest, non-electronic interactions must be also included. Such contributions, however, are usually very small. When using the code DOSRZ in the default mode, relaxation processes in the shell of atoms after photon interactions are taken into account only. We will denote that part of absorbed dose with D_{RLX}. Then the following partition of D^{MC} can be made:

$$D^{MC} = D_{E>ECUT} + D_{E\leqslant ECUT} + D_{RLX} \tag{2}$$

where ECUT corresponds to the above mentioned cutoff energy parameter Δ, $D_{E>ECUT}$ and $D_{E\leq ECUT}$ are the main contributions to absorbed dose due to interactions of electrons with energies $E > ECUT$ and $E \leq ECUT$, respectively. Note: in the MC codes the energy of electrons is expressed as total energy, i.e. including the rest mass energy, *rm*, of electrons, whereas the energy in the fluence distribution refers to the kinetic energy. Thus, the MC cutoff energy parameter ECUT corresponds to $\Delta+rm$. The expected equivalence between MC calculated SAN cavity integral and absorbed dose is then expressed by:

$$CI_{SAN}^{MC} \equiv D^{MC} - D_{RLX}. \tag{3}$$

2.2. Formalism

Frequently the following expression is used for the SAN cavity integral in a water volume with density ρ (ICRU 2016, Kumar *et al* 2016, Andreo *et al* 2017):

$$\mathrm{CI_{SAN}} = \int_{\Delta}^{\mathrm{E_{max}}} \Phi_{E,w}(E)\,\frac{L_{\Delta,\mathrm{w}}(E)}{\rho}\mathrm{d}E + \Delta\cdot\Phi_{E,w}(\Delta)\,\frac{S_{\mathrm{el,w}}(\Delta)}{\rho} \tag{4}$$

where $\Phi_{E,w}$ is the total electron fluence differential in kinetic energy E, $L_{\Delta,\mathrm{w}}$ the linear energy transfer (LET), and $S_{\mathrm{el},w}/\rho$ is the unrestricted mass electronic stopping power. The first expression at the right hand side is the fluence term FT as addressed above. The expression $\Phi_{E,w}(\Delta)\,\frac{S_{\mathrm{el},w}(\Delta)}{\rho}$ used in the second term, the track-end term TE, is a substitute for the number of electrons per unit mass which are going to drop down below Δ in their slowing down process. Note that the energy parameter Δ as used in equation (4) has a dual purpose: (a) it is the separation or cutoff parameter between the high and low energy group (and therefore the lower limit in the fluence integral), (b) it is the upper limit of energy transfer involved in the LET, L_{Δ}.

However, equation (4) is not complete when applied at a mixed photon/electron field. In order to represent the total dose, it-should comprise the energy dissipation of all electrons. This is not the case with the track-end term of equation (4). Whereas by far the largest number of energy losses of electrons with $E \leq \Delta$ occurs indeed at $E = \Delta$, and thus their contribution is correctly taken into account by the expression $\Delta\cdot\Phi_{E,w}(\Delta)\,\frac{S_{\mathrm{el},w}(\Delta)}{\rho}$, the contribution of electrons with $E < \Delta$ is missing. However, secondary electrons created in the scoring volume can indeed have an initial energy below Δ, for instance due to incoherent scattering (Compton interaction), photo-electric absorption, or bremsstrahlung processes. One may argue that the energy dissipation involved with that is insignificant. Nevertheless, the following points speak in favor of including contributions with $E < \Delta$ into a general expression for the SAN cavity integral:

(1) Such contributions may be small in a given situation, but it is not for sure whether this generally applies.

(2) Such contributions are taken into account in MC dose calculations. Therefore, they also should be taken into account in the MC calculation of the SAN cavity integral, in particular when both are compared.

(3) A standardized definition for the SAN cavity integral currently does not yet exist.

Thus, from a principal point of view, the following expression for the SAN cavity integral should be given preference:

$$\mathrm{CI_{SAN}} = \int_{\Delta}^{\mathrm{E_{max}}} \Phi_{E,w}(E)\,\frac{L_{\Delta,\mathrm{w}}(E)}{\rho}\mathrm{d}E + \mathrm{TE} \tag{5}$$

where TE is a more general denotation of the track-end term consisting of two parts, namely of $\mathrm{TE}_{E=\Delta}$ and $\mathrm{TE}_{E<\Delta}$.

2.3. Numerical considerations

In order enable a direct comparison of our numerical results with those in the paper by Kumar *et al* (2016), we used the same calculation tools and methods, i.e. (a) the codes FLURZ and DOSRZ from the EGSnrc system, and (b) same beam geometry and scoring volume which refer to a parallel beam of incident monoenergetic photons with 1 MeV with a circular cross-section with a radius of 5 cm, and a scoring volume of cylindrical shape, with radius 2 cm, height 1 cm and with its front face perpendicular to the beam direction at a depth of 2 cm in a cubic water phantom of 30×30 × 30 cm^3. However, our MC codes of FLURZ and DOSRZ, applied as FORTRAN codes, have been slightly modified in order to improve some of the approximations involved in the original versions. Details are given below.

Fluence term:

In MC calculations, the fluence of electrons differential in (kinetic) energy is usually calculated as the sum of all partial pathlengths in a given energy bin divided by the volume of the scoring volume (Kellerer 1971, Chilton 1978, Papiez and Battista 1994). Partial pathlengths are obtained according to the selected binning scheme. We have used a linear scheme with a bin width of 1 keV. In FLURZ the pathlength is assigned to the energy bins using the approximation that the restricted stopping power is constant during the step (Rogers *et al* 2025). We have implemented the more exact approximation that

the inverse of the LET is a linear function of energy (Hartmann and Andreo 2019). The consequences of these different approximations with respect to the algorithm for the fluence calculation are described in detail in the appendix.

There are further minor code depending issues influencing the calculation of fluence. For example, a bug which caused an artifact in the first energy bin was fixed only in 2017. A second example refers to the numerical value of rest mass energy, *rm* which have been reduced from a value slightly larger than 0.511 to the currently correct value slightly lower than that. Thus, the value used for the cutoff energy parameter Δ (which is $ECUT - rm$) was depending on the code version of FLURZ. In our version, the value of the kinetic energy Δ was set independently from *rm*.

DOSRZ also provides an output of the difference between the total dose and the dose only due to stoppers denoted as '*T*–*S*' (Rogers *et al* 2025). One may consider this output as the value of $D_{E>\mathrm{ECUT}}$ and thus also of the fluence term of equation (5). However, this is not exactly true. In case that after a transport step j the energy E_{j+1} is less than ECUT, the total energy loss $\mathrm{d}E = E_j - E_{j+1}$ is put into the contribution of $D_{E>\mathrm{ECUT}}$. The exact procedure, however, would require that the energy difference $E_j - \mathrm{ECUT}$ only is put into $D_{E>\mathrm{ECUT}}$, whereas the remaining energy (which is $ECUT - E_{j+1}$) is to be put into $D_{E\leq\mathrm{ECUT}}$. This inexactness has no influence on the calculation of total absorbed dose. However, it matters for the correct differentiation between the fluence term and the track-end term.

Track-end term:

Kumar *et al* (2016) calculated the main part of the track-end term, namely $\mathrm{TE}_{E=\Delta}$. based on the expression $\Delta{\cdot}\Phi_{E,w}(\Delta)\frac{S_{\mathrm{el},w}(\Delta)}{\rho}$. However, FLURZ does not provide the value of $\Phi_{E,w}$ exactly at Δ. In order to overcome this problem, they used a logarithmic energy binning in FLURZ which allows the calculation of fluence $\Phi_{E,w}$ very close to Δ. Even then another problem arises: the specific method used in FLURZ for the calculation of fluence is not very well appropriate to calculate the fluence at small energies. The reason is that $\Phi_{E,w}$ depends on further MC input parameter such as MAXIB (the number of bins) or ESTEPE (which controls the maximum fraction of energy loss in a transport step). This dependence leads to some ambiguity for the determination of the fluence close to Δ. A more reliable approach to determine $\mathrm{TE}_{E=\Delta}$ consists of the following simple method: the number of electrons just with the energy Δ can be derived by counting the number of such transport steps j where the electron energy E_j before the transport step is higher than ECUT and lower after that, multiplied by Δ. This method was implemented in our FORTRAN version of the DOSRZ and FLURZ codes. Note: Based on this approach, the value of $\Phi_{E,w}(\Delta)$ can be indeed simply derived from n_{m} which is the number of such steps per mass of the scoring volume by:

$$\Phi_{E,w}(\Delta) = n_{\mathrm{m}} / \left(\frac{S_{\mathrm{el},w}(\Delta)}{\rho}\right). \tag{6}$$

The output value '*T*–*S*' may be considered an alternative approach to determine the total track-end term TE (Kumar *et al* 2023). However, due to the inexactness of splitting the energy losses at ECUT into the contributions of $D_{E>\mathrm{ECUT}}$ and $D_{E\leq\mathrm{ECUT}}$, this approach is less accurate. In our version of DOSRZ, the number of stoppers at $E = \Delta$ and $E < \Delta$ are separately scored and provided as further output values.

3. Results

Values of $\mathrm{FT}^{\mathrm{MC}}$ and $\mathrm{TE}^{\mathrm{MC}}$ have been calculated using the code modifications addressed above in dependence of different combinations of the MC transport parameters ECUT and Δ (subsequently expressed in terms of kinetic energy in keV). The fluence term $\mathrm{FT}^{\mathrm{MC}}$ was determined based on fluence results obtained with our modified version of the fluence subroutine. The cutoff energy ECUT was set exactly $\Delta + rm$. Results of the components of the SAN cavity integral were given in percent values relative to the MC calculated absorbed dose. In table 1, values for Δ (and thus for ECUT) were set at 1, 2, 5, 10, 20, 50, and 100 keV. The MC code FLURZ also allows to input different values for Δ and ECUT. In this case Δ refers to the energy parameter of L_{Δ} only. In table 2, Δ was kept constant at 1 keV whereas ECUT was varied the same way as in table 1. The transport parameter AP for photons was always set at 1 keV.

Table 1. Dependence of various MC calculated results on Δ, (and ECUT equal to Δ): absorbed dose D per incident photon fluence (in units of Gy cm^{-2}), the SAN cavity integral CI_{SAN}, the fluence term FT and the track-end term TE differentiated into the components $TE_{E=ECUT}$ and $TE_{E<ECUT}$ in % of absorbed dose. For comparison with the fluence term FT, the partial absorbed dose provided by DOSRZ, D_{T-S} was added as last column.

				TE		
Δ (= *ECUT*) keV	*D*Gy cm^2	CI_{SAN}%	FT%	$TE_{E=ECUT}$ %	$TE_{E<ECUT}$ %	D_{T-S} %
1	4.861×10^{-12}	99.96	92.60	7.36	0.00	92.84
2	4.859×10^{-12}	99.96	92.92	7.02	0.01	93.06
5	4.860×10^{-12}	99.96	92.86	7.05	0.05	93.06
10	4.862×10^{-12}	99.96	92.09	7.73	0.14	92.20
20	4.863×10^{-12}	99.96	90.03	9.51	0.42	90.15
50	4.863×10^{-12}	99.96	83.38	14.94	1.64	83.64
100	4.859×10^{-12}	99.96	72.92	22.71	4.33	73.16

Table 2. Dependence of various MC calculated results on ECUT (where Δ always was kept at 1 keV): absorbed dose D per incident photon fluence (in units of Gy cm^2), the SAN cavity integral CI_{SAN}, the fluence term FT and the track-end term TE differentiated into the components $TE_{E=ECUT}$ and $TE_{E<ECUT}$ in % of absorbed dose. For comparison with the fluence term FT, the partial absorbed dose provided by DOSRZ, D_{T-S} was added as last column.

				TE		
ECUT (keV) Δ= 1 keV	*D* Gy cm^2	CI_{SAN} %	FT %	$TE_{E=ECUT}$ %	$TE_{E<ECUT}$ %	D_{T-S} %
1	4.860×10^{-12}	99.96	92.60	7.36	0.00	92.84
2	4.863×10^{-12}	99.96	87.66	6.86	5.43	88.55
5	4.859×10^{-12}	99.96	81.62	6.42	11.91	82.39
10	4.861×10^{-12}	99.96	77.12	6.68	16.15	77.87
20	4.864×10^{-12}	99.96	72.16	7.81	19.98	72.98
50	4.863×10^{-12}	99.96	63.52	11.58	24.86	64.58
100	4.860×10^{-12}	99.96	53.77	16.95	29.23	55.05

4. Conclusion

The results presented in tables 1 and 2 clearly demonstrate:

(1) If the electron fluence is calculated avoiding the approximations made in FLURZ, the ratio between the SAN cavity integral and absorbed dose is exactly given by:

(2) This statement does not depend on the specific choice of the MC transport parameter ECUT and Δ.

(3) However, the specific contributions of partial components indeed depend on these parameters.

(4) Although equation (7) was verified for the example used in this paper only, the principal consideration addressed in the methods section strongly suggests that the conclusions above generally applies.

$$\frac{CI_{SAN}}{D} = \frac{1 - D_{RLX}/D}{1}. \tag{7}$$

In the example of a 1 MeV photon beam used in this paper, the relative contribution of D_{RLX} to total dose D was less than 0.05%. The contribution may increase with decreasing energy of incident photons (Hartmann *et al* 2021).

Admittedly, not all fine numerical details involved in the MC codes FLURZ and DOSRZ are made sufficiently transparent in the available user manuals. However, comparisons of numerical results must take into account such fine details when aiming at meaningful interpretations of the relation between the SAN cavity integral and absorbed dose. Our aim was to contribute to that.

Acknowledgment

We particularly wish to thank Ernesto Mainegra-Hing from the National Research Council Canada for providing valuable information on the MC code FLURZ of the EGSnrc system.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary information files).

Appendix: Formalism to calculate the fluence of electrons differential in energy.

The calculation of the fluence of electrons differential in (kinetic) energy in a volume can be based on the determining the sum of partial pathlengths within a given energy bin divided by that volume. This approach requires the segmentation of a total pathlength i of an electron into partial pathlengths $l_{i,j}$. according to a selected binning scheme with single energy bins j. The relation between energy E and the pathlength of electrons slowing down in energy through sub-threshold processes is generally given as:

$$l = \int_{E_2}^{E_1} \frac{1}{L_\Delta(E')} \mathrm{d}E' \tag{A1}$$

where E_1 and E_2 with $E_1 > E_2$ are the start and stop energy of an electron path, and $L_\Delta(E)$ is the linear energy transfer with respect to Δ. The formula is applicable for the calculation of the total pathlength where E_1 and E_2 refer to the start and stop energy of a total path length, as well as for the partial pieces where E_1 and E_2 refer to the top and bottom energy of a specific bin, E_j and E_{j+1}.

Depending on the approximation used for the LET as a function of energy, equation (A1) leads to different expressions for the calculation of the fluence spectrum.

Approximation 1: The LET is constant along the total pathlength
This approximation is good for higher energy electrons but is increasingly poor for low-energies (Rogers *et al* 2025). The integral of equation (A1) goes over into:

$$l_i = \frac{\mathrm{d}E_i}{L_{\Delta,i}}. \tag{A2}$$

The total energy loss $\mathrm{d}E_i$ between $E_{i,1}$ and $E_{i,2}$ can be segmented according to the energy binning as:

$$\mathrm{d}E_i = \left(E_{i,2} - E_{i,j(E_2)}\right) + \sum_{i,j(E_1)}^{i,j(E_2)} \left(E_{i,j+1} - E_{i,j}\right) + \left(E_{i,j(E_1)} - E_{i,1}\right). \tag{A3}$$

The meaning of the different energies are exemplified in the figure A1 below.

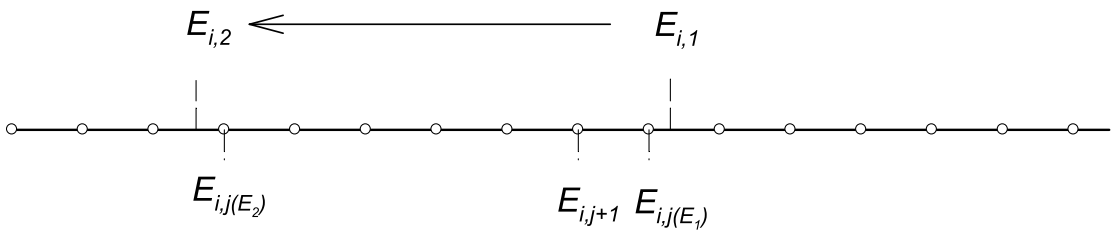


Figure A1. Start energy $E_{i,1}$ and stop energy $E_{i,2}$ of an electron path i, and top and bottom energy, E_j and E_{j+1} of a single bin j. $E_{i,j(E_1)}$ is the top energy of the next lower bin with respect to $E_{i,1}$, and $E_{i,j(E_1)}$ is the bottom energy of the next higher bin with respect to $E_{i,2}$.

It then follows that the partial path length l_{ij} between $E_{i,j}$ and $E_{i,j+1}$ is given by:

$$l_{ij} = \frac{\mathrm{d}E_{i,j}}{L_{\Delta,i}} = l_i \frac{E_{i,j+1} - E_{i,j}}{\mathrm{d}E_i} \tag{A4}$$

where $E_{i,j+1} - E_{i,j}$ is one of the energy intervals between $E_{i,1}$ and $E_{i,2}$ of equation (A3). Equation (A4) is the method implemented in the code FLURZ.

Approximation 2: The inverse of the LET is a linear function of energy between top and bottom energy of a single bin.

This approximation for $L_{\Delta,i}$is similar to that described in the manual PIRS-898, EGSnrc C++ class library (2021). It is particularly more realistic by reflecting the fact that $L_{\Delta,i}$ is a function of energy. Here it is approximated as:

$$\frac{1}{L_{\Delta,i}(E)} = a + bE. \tag{A5}$$

The integral of equation (A1) goes over into:

$$l_{ij} = \int_{E_{i,j}}^{E_{i,j+1}} (a + bE')\,\mathrm{d}E' = \left(E_{i,j+1} - E_{i,j}\right)\left(a + b\left(\frac{E_{i,j+1} + E_{i,j}}{2}\right)\right). \tag{A6}$$

which leads to:

$$l_{ij} = \frac{E_{i,j+1} - E_{i,j}}{L_{\Delta,i}\left(\frac{E_{i,j+1}+E_{i,j}}{2}\right)}. \tag{A7}$$

The algorithm using equation (A7) is implemented in our modified code version of FLURZ.

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